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Curved dilatonic brane-worlds and the cosmological constant problem

Natxo Alonso-Alberca[1], **Bert Janssen**[2] **and Pedro J. Silva**[3]

Instituto de Física Teórica, C-XVI,
Universidad Autónoma de Madrid
E-28006 Madrid, Spain

ABSTRACT

We construct a model for dilatonic brane worlds with constant curvature on the brane, i.e. a non-zero four-dimensional cosmological constant, given in function of the dilaton coupling and the cosmological constant of the bulk. We compare this family of solutions to other known dilatonic domain wall solutions and apply a self-tuning mechanism to check the stability of our solutions under quantum fluctuations living on the brane.

[1]E-mail address: `natxo.alonso@uam.es`
[2]E-mail address: `bert.janssen@uam.es`
[3]E-mail address: `psilva@delta.ft.uam.es`

Recently the idea of brane worlds has received a lot of attention. In this picture our space-time has one (or more) extra non-compact spacial dimensions. In the scenario of Randall and Sundrum [1, 2], a three-brane in a five-dimensional AdS-space was constructed, confining a Standard Model-like gauge theory on the brane. Due to the warped form of the metric of the solution, a different approach to the hierarchy problem was presented. But it also turns out that in this picture, perturbations of the metric on the brane have a five-dimensional profile which is normalisable and localised to the brane as if it were four-dimensional. Hence, an observer living in this brane world would see both gauge as gravity physics in the same way as an observer in a four-dimensional space-time.
Different generalisations of this picture including a dilaton field were given in [3]-[7] while on the other hand, generalizations to non-flat brane worlds (without dilaton) appeared in [8], where a four-dimensional cosmological constant was introduced via constant curvature branes. Here it was shown that, in the brane world scenario, the four-dimensional cosmological constant is a geometrical property of the three-brane (namely its internal curvature), which in principle can be independent of the five-dimensional one.
This shed new light on an old problem, namely why the observed cosmological constant in our universe is so small. One would expect that the non-zero vacuum expectation value of Standard Model fields would generate a non-zero vacuum energy, which would result in an effective non-zero cosmological constant.[4] Now that it turns out that in the brane world picture the four-dimensional cosmological constant is a geometrical property of the brane, one should ask how this property is affected by quantum fluctuations of the field theory living on this brane.
In [4, 5, 7] it was observed that fluctuations of the brane tension, due to quantum corrections of the field theory living on a (flat) dilatonic brane, do not generate a four-dimensional cosmological constant. Via a self-tuning mechanism, the fluctuations in the brane tension can be absorbed into shifts of the dilaton, such that the quantum corrections do not curve the brane itself or the extra dimension. Thus, starting with flat brane worlds, no extra curvature, and hence no four-dimensional cosmological constant, is generated.
Recent astronomical observations, however, point in the direction of a small positive, but non-zero cosmological constant. It is therefore interesting to look at brane-world models of non-zero curvature and see if there is a self-tuning mechanism working in these cases, which could protect this curvature from quantum corrections. The aim of this letter is to combine the results of [3]-[7] and [8], constructing dilatonic brane-worlds with a non-zero four-dimensional cosmological constant and test the self-tuning mechanism on these solutions. The organisation of this letter is as follows: we start with the construction of the curved dilatonic brane-world solutions and then give a small discussion of these solutions, comparing the obtained solutions with a class of domain wall solutions given in [10]. Finally we will analyse the self-tuning mechanism in the case of curved dilatonic branes.

As a starting point let us consider the action of five-dimensional dilatonic gravity coupled

[4]For a more general discussion on the cosmological constant problem, see [9] and references therein.

to a brane source in the presence of a (five-dimensional) cosmological constant:

$$
\begin{aligned}
S &= \frac{1}{\kappa}\int d^4x\ dy\ \sqrt{|\hat{g}|}\ \Big[\hat{R}+\tfrac{4}{3}(\partial\phi)^2 - e^{\frac{\alpha}{3}\phi}\Lambda\Big] \\
&\qquad\qquad - \int d^4x\ \sqrt{|\bar{g}|}\ e^{\frac{\beta}{3}\phi}\ V_0\ , \qquad (1)
\end{aligned}
$$

where V_0 is the tension of the brane source and $\bar{g}_{mn} = \hat{g}_{\mu\nu}\delta^\mu_m\delta^\nu_n$ the induced metric on the brane. To solve the equations of motion, we propose the following curved brane-world Ansatz:

$$
\begin{aligned}
ds^2 &= a(y)^2\tilde{g}_{mn}dx^m dx^n - dy^2 \\
\phi &= q\log a(y) + \phi_0\ , \qquad (2)
\end{aligned}
$$

with q and ϕ_0 arbitrary constants. The internal brane metric $\tilde{g}_{mn}$ depends only on the internal coordinates x^m. Plugging this Ansatz in the equations of motion of the action (1) gives the following set of differential equations:

$$
\begin{aligned}
&\tilde{R}_{mn} - \tilde{g}_{mn}\Big[a\ \ddot{a} + 3\ \dot{a}^2 + \tfrac{1}{3}\ a^{\frac{\alpha q}{3}+2}e^{\frac{\alpha}{3}\phi_0}\Lambda\Big] = +\tfrac{1}{6}\kappa V_0\ a^{\frac{\beta q}{3}+2}\ e^{\frac{\beta}{3}\phi_0}\tilde{g}_{mn}\delta(y)\ , \\
&4\ a^{-1}\ddot{a} + \tfrac{4}{3}\ q^2 a^{-2}\dot{a}^2 + \tfrac{1}{3}\ a^{\frac{\alpha q}{3}}e^{\frac{\alpha}{3}\phi_0}\Lambda = -\tfrac{2}{3}\kappa V_0\ a^{\frac{\beta q}{3}}\ e^{\frac{\beta}{3}\phi_0}\delta(y)\ , \qquad (3) \\
&q\ a^{-1}\ddot{a} + 3\ q\ a^{-2}\dot{a}^2 - \tfrac{\alpha}{8}\ a^{\frac{\alpha q}{3}}e^{\frac{\alpha}{3}\phi_0}\Lambda = +\tfrac{\beta}{8}\kappa V_0\ a^{\frac{\beta q}{3}}\ e^{\frac{\beta}{3}\phi_0}\delta(y)\ ,
\end{aligned}
$$

where $\tilde{R}_{mn}$ is the Ricci tensor of the internal metric $\tilde{g}_{mn}$ and a dot denotes derivative with respect to y. Analogous as in [8], the first equation of (3) can only be satisfied if the y-dependence vanishes:

$$
a\ \ddot{a} + 3\ \dot{a}^2 + \tfrac{1}{3}\ a^{\frac{\alpha q}{3}+2}e^{\frac{\alpha}{3}\phi_0}\Lambda + \tfrac{1}{6}\kappa V_0\ a^{\frac{\beta q}{3}+2}\ e^{\frac{\beta}{3}\phi_0}\delta(y) = \tilde{\Lambda}\ , \qquad (4)
$$

with $\tilde{\Lambda}$ an abritrary integration constant which we interpret as the four-dimensional cosmological constant in the brane-world, since the first equation of (3) now translates into $\tilde{R}_{mn} = \tilde{\Lambda}\ \tilde{g}_{mn}$. The solution to the equations (3)-(4) is given by

$$
\begin{aligned}
ds^2 &= \Big[1 - \tfrac{\alpha}{12}\ \sqrt{-\Lambda}\ |y|\Big]^2\ e^{\frac{\alpha}{3}\phi_0}\ \tilde{g}_{mn}dx^m dx^n - dy^2\ , \\
\phi &= -\frac{6}{\alpha}\ \log\Big[1 - \tfrac{\alpha}{12}\ \sqrt{-\Lambda}\ |y|\Big]\ , \qquad (5)
\end{aligned}
$$

where the coordinate y runs from 0 to $\frac{12}{\alpha\sqrt{-\Lambda}}$ and the four-dimensional metric and cosmological constant satisfy

$$
\tilde{R}_{mn} = \tilde{\Lambda}\ \tilde{g}_{mn}\ , \qquad\qquad \tilde{\Lambda} = \tfrac{16-\alpha^2}{48}\Lambda\ e^{\frac{\alpha}{3}\phi_0}\ . \qquad (6)
$$

In the conformal frame, the above solution takes the form

$$
\begin{aligned}
ds^2 &= \exp\Big(\tfrac{\alpha}{6}\ \sqrt{-\Lambda}\ |z|\Big)\Big[e^{\frac{\alpha}{3}\phi_0}\tilde{g}_{mn}dx^m dx^n - dz^2\Big]\ , \\
\phi &= -\tfrac{\sqrt{-\Lambda}}{2}\ |z|\ . \qquad (7)
\end{aligned}
$$

The brane tension and dilaton coupling in the source term are given in terms of α and Λ by the jump equations:

$$V_0 = \frac{\alpha}{2\kappa}\sqrt{-\Lambda}\ , \qquad\qquad \beta = \frac{8}{\alpha}\ . \tag{8}$$

We thus see that the brane world are surfaces of positive (dS) or negative (AdS) constant curvature, depending on the dilaton coupling α. It is remarkable that in the limit $\tilde{\Lambda} \to 0$ ($\Lambda \neq 0$), we do not recover the general "flat" dilatonic RS theory [3, 4, 6], but only a particular case $\alpha = \pm 4$.[5] Therefore, in contrast to "flat" dilatonic RS theory, it is no longer possible to make contact with the original RS scenario [1, 2]. Nor is it possible, in the limit $\alpha \to 0$ (ϕ trivial), to make contact with the solutions of [8], as can clearly seen in the conformal frame (7). The solution for $\Lambda = 0$ ($\tilde{\Lambda} \neq 0$) was given in [7].

It is interesting to compare the solution (5) with the dilatonic domain wall solutions found in [10]. It is clear that (5) can not be identified as one of the solutions given in [10]. This is due to the fact that the Ansatz used in [10] is different as our Ansatz (2). The main difference lays in the fact that the Ansatz of [10] considers branes with constant spatial curvature, while the Ansatz (2) constructs branes with constant world volume (space-time) curvature. Still, there exists a particular solution that belongs to both classes, i.e. that can be obtained from each class as a special limit. For this case, the domain wall has to be flat, which implies $\alpha = \pm 4$ in equation (5) and $M = k = 0$ in the Type II solutions of [10]. Furthermore, in the Ansatz of [10] the metric components g_{tt} and g_{ij} (the radial function in front of the spatial part of the brane world volume) should be identified. This particular solution is (in our notation, for $\alpha = 4$ e.g.):

$$\begin{aligned} ds^2 &= \left(1 - \tfrac{1}{3}\sqrt{-\Lambda}\ |y|\right)^2 e^{\frac{4}{3}\phi_0}\left[dt^2 - dx_i^2\right] - dy^2\ , \\ e^{\phi} &= \left(1 - \tfrac{1}{3}\sqrt{-\Lambda}\ |y|\right)^{-3/2}\ . \end{aligned} \tag{9}$$

An analysis, similar as the one done in [10] reveals that this solution is a static domain wall, due to the fact that for this case the potential is constant. This particular solution (9) was first given in [11].

A straightforward analysis of the perturbations of the metric (5) gives the following profile for the four-dimensional graviton

$$\psi(y) = \left[1 - \tfrac{\alpha}{12}\sqrt{-\Lambda}\ |y|\right]^2\ , \tag{10}$$

which is normalizable in the interval $[0, \frac{12}{\alpha\sqrt{-\Lambda}}]$ and localized around $|y| = 0$. Note that the dependence on dilaton coupling α is much weaker than in the "flat" dilatonic RS scenario, where we had an exponential dependence on the coupling.

[5] In [6] the case $\alpha = 4$ was erroneously identified with the case $\Lambda = 0$ due to a coordinate singularity in the conformal frame. The holographic frame solution however is completely regular and can be seen as the limit of (5) for $\tilde{\Lambda} = 0$.

Finally, note that, similar to other dilatonic brane world models, the solution (5)-(8) has a time-like naked singularity in $y = \frac{12}{\alpha\sqrt{-\Lambda}}$ $(z = \infty)$.

In the brane world picture, the four-dimensional cosmological constant $\tilde{\Lambda}$ is, rather than a input parameter of the Lagrangian, a geometrical quantity, related to the curvature of the domain wall. In [4, 5, 7] it was argued that for flat domain walls (i.e. $\tilde{\Lambda} = 0$), the quantum fluctuations of the gauge theory living on the brane can be absorbed in shifts of the dilaton, which turns out to be a symmetry of the solutions. This mechanism, called self-tuning, makes that no extra curvature is generated and that the four-dimensional cosmological constant remains zero, even after quantum corrections.
Since recent astronomical observations seem to indicate that the cosmological constant might be very small, but non-zero and positive, a natural question to ask is how far this self-tuning mechanism can be extended, in particular to branes with non-zero curvature (i.e. solutions with non-zero four-dimensional cosmological constant). It turns out that no self-tuning mechanism is possible for the solutions we have constructed, due to the fact that the jump equation (8) does not depend explicitly on ϕ_0. In fact, V_0 only depends on the bulk parameters and an arbitrary integration constant (which, without lost of generality, we set equal to $\frac{12}{\alpha\sqrt{-\Lambda}}$), indicating the position of the naked singularity. Changes of V_0 due to quantum fluctuations could only be absorbed by this integration constant, but this would mean that the positions of the singularities should change, which seems to have no sense. This seems to indicate that the self-tuning mechanism of [4, 5, 7] is only valid for the case of flat branes and does not provide a satisfying explanation of the cosmological constant problem in the light of the recent astronomical observations.

The family of solutions we have constructed does not include the $\alpha = 0$ case, as can be seen from (5) and (8). With the Ansatz $\phi = \phi(y)$, the equations of motion for $\alpha = 0$ read:

$$\begin{aligned} &\ddot{\phi} + 4\,a^{-1}\dot{a}\dot{\phi} = 0, \\ &a^{-1}\ddot{a} + \tfrac{1}{3}\,\dot{\phi}^2 + \tfrac{1}{12}\Lambda = 0, \\ &a^{-1}\ddot{a} + 3\,a^{-2}\dot{a}^2 + \tfrac{1}{3}\Lambda = \tilde{\Lambda} a^{-2}. \end{aligned} \tag{11}$$

The first of these equations gives

$$\dot{\phi} = \gamma a^4\,, \tag{12}$$

where γ is an arbitrary integration constant. Substituting (12) in (11), the last two equations reduce to a single one, given by:

$$\dot{a} = \epsilon\,\sqrt{\frac{\gamma^2}{9}\,a^{-6} - \frac{\Lambda}{12}\,a^2 + \frac{\tilde{\Lambda}}{3}}\,, \tag{13}$$

where $\epsilon = \pm 1$ determines the branch of the square root chosen in the solution. The sign of the argument of the square root in (13) will depend on the values of Λ and $\tilde{\Lambda}$. Both Λ and $\tilde{\Lambda}$ are arbitrary and independent. The solution only makes sense when this argument is positive. Integrating (13) we obtain:

$$\int^a \frac{\epsilon\, da'}{\sqrt{\frac{\gamma^2}{9}\, a'^{-6} - \frac{\Lambda}{12}\, a'^2 + \frac{\tilde{\Lambda}}{3}}} = y + y_0\,. \tag{14}$$

The l.h.s. of (14) cannot be expressed in terms of any elementary function, so it is hard to see whether a self-tuning mechanism could work. As in [7], it could be possible to study the bounds on the brane cosmological constant depending on the signs and values of Λ and $\tilde{\Lambda}$. We leave this question open for future research.

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